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EXPERIMENTAL STUDY AND NUMERICAL ANALYSIS ON VIBRATION CHARACTERISTICS OF SIMPLY SUPPORTED OVERHANG BEAM UNDER LARGE DEFORMATION

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ABSTRACT

The present paper undertakes an experimental and theoretical dynamic analysis of a simply supported overhang beam. A set up, consisting of two knife edges to simulate simply supported boundary conditions and provision for application of transverse load through two suspended weight pans at the extremities of the beam, is prepared to carry out experimentations on a slender beam. The free vibration experimentation is carried out by exciting the loaded system with the blow of a soft rubber hammer and capturing the response by a miniature accelerometer. The theoretical dynamic analysis is carried out on the basis of energy principles and a technique based on the developed axial forces is introduced to incorporate the stretching effect in the present work. The simple supports are replaced by two springs so that the point above the support can have some movement depending on the stiffness of the springs.

Keywords: Overhang Beam, Simply Supported Beam, Free Vibration.

1. INTRODUCTION

Slender beams are one of the most common structural elements that can be used separately or in association with other beams or plates, in many fields of engineering. They are extensively used in various branches of modern civil, mechanical and construction engineering. In these applications they are regularly subjected to static and dynamic loads. Hence, analysis of beams under different loading and boundary conditions has always been an area of immense interest to researches and research studies carried out in this field have been recorded in different review papers.

Sathyamoorthy [1] reviewed the works on classical methods of non-linear (geometric, material and other type of non-linearities) beam analysis. Sathyamoorthy [2] also surveyed the developments on non-linear beam analysis under static and dynamic conditions using finite elements methods. Kapania and Raciti [3] presented a review on advances in the analysis of laminated structures (beams and plates) using shear deformation theories, finite elements methods and also on buckling of such structures. Mei [4] presented a finite element method to determine the nonlinear frequency of beams for large amplitude free vibrations. Kim and Dickinson [5] performed a simple analysis of the free vibration problem of slender beams subject to various complicating effects using Rayleigh-Ritz method, with orthogonally generated polynomials as admissible functions. Klausbruckner and Pryputniewicz [6] put forward a theoretical and experimental study of coupled vibrations of channel beams. Their experimental analysis was based on laser hologram interferometry. Ganapathi et al [7] studied the nonlinear free flexural vibrations of orthotropic straight and curved beams utilizing a cubic B-spline shear flexible curved element, based on the field consistency principle. They solved the nonlinear governing equations by employing Newmark's numerical integration scheme coupled with modified Newton-Raphson iteration technique.

Azrar et al [8] developed a semi-analytical approach to the nonlinear dynamic response problem of simply supported and clamped beams based on Lagrange's principle and the harmonic balance method. Kapuria et al [9] carried out static and free vibration response of layered FGM beams experimentally as well as theoretically. Holland et al [10] described the behavior of a slender, tapered, cantilever beam loaded through a cable attached to its free end. Large static deflections were computed together with natural frequencies and mode shapes for small-amplitude vibrations about equilibrium. Gupta et al [11] investigated large amplitude free vibration analysis of uniform, slender and isotropic beams through a relatively simple finite element formulation, applicable to homogenous cubic nonlinear temporal equation. This finite element formulation was applied to analyze free vibration of uniform isotropic Timoshenko beams with geometric nonlinearity by Gunda et al [12]. Karaagac et al [13] presented theoretical and experimental free vibration studies of a slender cantilever beam with an edge crack. In this study, a finite element algorithm based on energy method was developed and experiments were carried out in order to validate the results obtained from the numerical method. Giunta et al [14] proposed a unified formulation of one-dimensional beam models for the free vibration analysis of functionally graded beams. It is evident from the review of existing literature that theoretical and experimental investigations of a single beam under different loading and boundary conditions have been carried out extensively.

The present paper puts forward an experimental and theoretical free vibration analysis of a simply supported overhang beam under transverse loading at its extreme ends. Fig. 1 shows such a simply supported thin rectangular beam having two overhang portions of equal length, with its significant dimensions and the coordinate system used in the present analysis. As can be seen from the figure, the span between the two supports is l and the two overhang portions are of equal length (a). The total length of the beam is represented by L, which is related to the span length through the relation, $L = l + 2a$. Such experiments on a beam have not been performed previously. Also a technique based on the developed axial forces is introduced to incorporate the stretching effect in the present work. Although the present work is limited within the elastic regime of material behaviour but the analysis method can be easily reconstructed for post elastic behaviour having multilinear material characteristics.

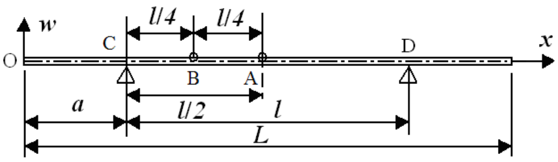

Fig 1. Simply supported overhang beam with significant dimensions and the coordinate system

2. EXPERIMENTATION

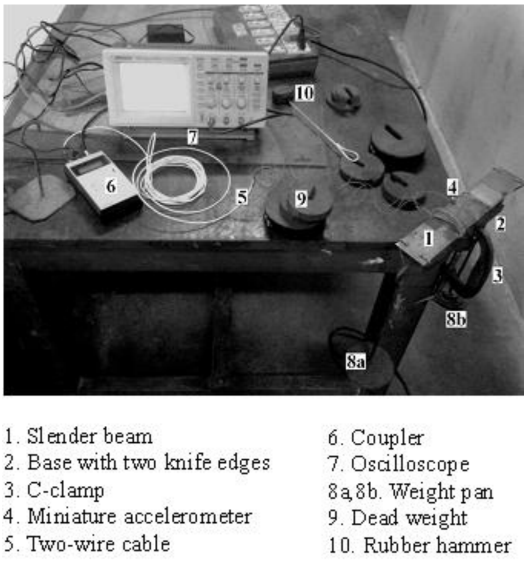

Fig 2. Photograph of the experimental set up

A basic experimental set up, consisting of two knife edges at a fixed distance apart that simulate simply supported conditions, is prepared to carry out free vibration experimentation on a slender beam. Figures 2 and 3 present the photograph and schematic diagram of the above mentioned experimental set up. The set up consists of a solid base with knife edges, an accelerometer, coupler and oscilloscope. The free vibration experimentation is carried out at the deflected configuration by exciting the system with the blow of a soft rubber hammer. The following section provides a brief description of the set up and test procedure.

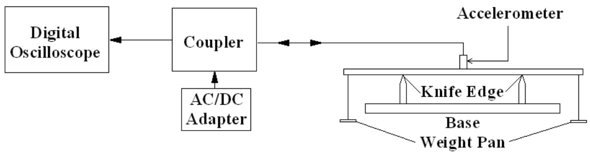

Fig 3. Schematic diagram of experimental setup for free vibration experiments on simply supported overhang beam.

2.1 Experimental Set-up

A slender beam with rectangular cross-section is carefully positioned over the two knife edges of the base in such a way that the overhang lengths at the two ends of the beam are equal. The base itself is rigidly fixed to a heavy base with C-clamps to provide further stability. Two weight pans are suspended from the extremities of the slender beam as loading arrangement for application of transverse load at those locations. A shear mode piezoelectric accelerometer (Manufacturer: Kistler Instrument Corporation, Type: 8728A500, acceleration range: ±500g (g = 9.80665 m/s^2), frequency range: 1 Hz–10 kHz (±5%)) is mounted on the beam at suitable locations using Petro-Wax adhesive. The positions of the accelerometer are carefully selected to avoid any nodal points. The mass of the accelerometer is 1.6 grams, which is significantly less than the mass of the beam. Hence it can be assumed that system response is not significantly altered by the effect of mass loading of the accelerometer. Constant current power supply to the impedence converter of the accelerometer is provided by a coupler (Manufacturer: Kistler Instrument Corporation, Type: 5114, Frequency response: 0.07 Hz – 60 kHz (±5%)) connected to the accelerometer. It also decouples the DC bias voltage from the output signal. The connection between the accelerometer and the coupler is through a two-wire cable. The coupler provides the electrical interface between the accelerometer and the display device, a digital storage oscilloscope (Manufacturer: Tektronix Inc., Model: TDS 210) with the following specifications: peak detect bandwidth: 50MHz, sample rate range: 50 samples/s–1Gigasamples/s, record length: 2500 samples, and lower frequency limit: 10 Hz. It has the capability to transform a time domain signal into frequency domain through a Fast Fourier Transform (FFT) module.

2.2 Test Procedure

Experiments are carried out to determine the free vibration characteristics of the slender beam at its deflected configuration under transverse loading. The set up is readied by making the electrical connections for oscilloscope, coupler and accelerometer. The accelerometer is mounted on the test specimen at a predefined location using adhesive. A two-wire cable between the accelerometer and the coupler is used and the signal and power share the same line. Output from the coupler is connected to one of the channels of the oscilloscope. Equal transverse load at the two ends of the beam is applied by placing dead weights on the weight pans attached at the extremities of the beam. Under loading the beam assumes a deflected configuration. The oscilloscope is set to 'math' mode and 'auto' trigger mode is kept on. It is then kept ready by pressing the 'RUN' button as the system is hammered to provide disturbance. Oscilloscope captures the signal from the vibrating beam and plots the signal in frequency domain. The natural frequencies of the beam at the deflected configuration can be called loaded natural frequency. The oscilloscope captures and plots the signal from the vibrating system in frequency-amplitude plane. Using vertical cursors frequency (in Hz) of the signal is read from the display and the data is tabulated. The procedure is repeated for the next load level after adding dead weight to the pan to gradually increase load.

3. THEORETICAL ANALYSIS

The theoretical dynamic analysis is carried out in two steps. First, deflection is statically imposed by applying transverse concentrated load on the two ends of the beam and then the free vibration analysis is performed as an eigen value problem to identify the natural frequencies of the system at deflected configuration. The static analysis yields the initial deflection profile, which is used in the subsequent free vibration analysis. As the dynamic problem is solved on the basis of the solution of the static displacement field, the stretching effect of statically imposed displacement is incorporated into the dynamic system.

The origin of the coordinate system is taken at the left end of the beam, as shown in figure 1. In the present work cross-section of the beam has been taken as rectangular and the width and thickness are denoted by *b* and *t*, respectively. It is assumed that the thickness of the beam is sufficiently small to ignore the effects of shear deformation. The mathematical formulation is further based on the assumption that beam material is isotropic, homogeneous and linearly elastic. It should be mentioned here that in the present formulation axial displacement is not considered directly and stretching is taken into account through axial forces. The mathematical formulation of both the static and dynamic analysis is based on energy formulation.

3.1 Static Analysis

It is known from the principle of minimum potential energy that, $\delta(U+V)=0$. (1)

Here, *V* is the work function or potential of the external forces and *U* is the total strain energy stored in the system. The expression of the strain energy can be derived from the following expression,

$$U=\frac{E}{2}\int_{vol}(\varepsilon_x)^2\,dV \qquad (2)$$

where, $\varepsilon_x=-z\,(d^2w/dx^2)$ is the axial strain of a fiber due to bending action. *w* denotes transverse displacements of mid-plane and *x* denotes axial coordinate. It is to be noted that all the computations are carried out in normalized coordinate ξ ($= x / L$), *L* being the overall length of the beam. However, in the present case, an extra term containing the axial force distribution (N_x) along the length of the beam is included in the total strain energy (*U*) expression. This term (N_x) takes into account the stretching effect arising out of the change in length of the beam. So, the complete expression of the strain energy (*U*) of the system is given as,

$$U=\frac{EI}{2L^3}\int_0^1\left(\frac{d^2w}{d\xi^2}\right)^2 d\xi+\frac{1}{2L}\int_0^1 N_x(\xi)\left(\frac{dw}{d\xi}\right)^2 d\xi \qquad (3)$$

where, *E* and *I* are the elastic modulus of beam material and moment of inertia of the cross-section, respectively.

The external concentrated loading of magnitude *P* is applied at the two extreme ends of the overhang beam to produce the bending deformation. The total potential energy for the applied external loading is given as follows.

$$V=P\,w|_{\xi=0}+P\,w|_{\xi=1} \qquad (4)$$

where, $w|_{\xi=0}$ and $w|_{\xi=1}$ are the deflections of the two extreme ends of the beam.

For the theoretical analysis the simple supports are replaced by two springs of stiffness *k*, as shown in figure 4. It implies that the point above the support can have some movement depending on the stiffness of the springs. This is done in order to replicate the actual situation prevalent during the free vibration analysis.

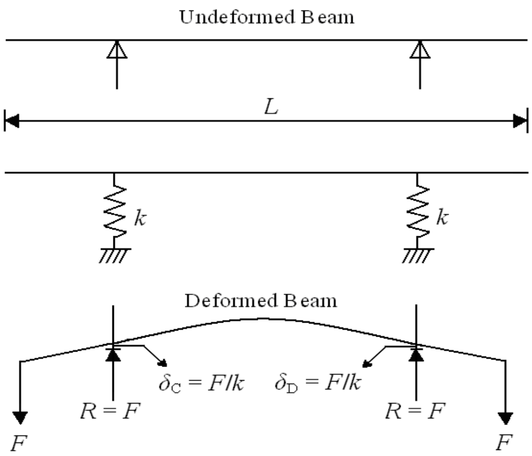

Fig 4. Schematic representation of undeformed and deformed beam.

The above mentioned energy functionals (*U* and *V*) can be determined from the assumed transverse (*w*) displacement field or which can also be called as the deflection. This displacement field can be approximated as linear combinations of orthogonal admissible functions and unknown coefficients d_i as shown below.

$$w(\xi)=\sum_{i=1}^{nw} d_i\,\phi_i\,(\xi) \qquad (5)$$

Here, $\phi(\xi)$ is a set of *nw* numbers of orthogonal functions, and these functions are to be selected in such a way that they satisfy the flexural boundary conditions of the beam. In order to generate the start function for the approximate displacement field a 5th order polynomial of the form shown in Eq. (6) is assumed and this equation is subjected to the geometric boundary conditions of the beam as mentioned below in Eq. (7).

Polynomial: $\phi(\xi) = \sum_{i=0}^{5} C_i \, \xi^i$, where, $C_0 = 1$ (6)

Boundary Conditions:

$$\frac{d^2 w}{d\xi^2} = 0 \ \text{ at } \xi = 0 \text{ and } 1,$$

$$w = F/k \text{ at } \xi = a/L \text{ and } (1 - a/L) \tag{7}$$

$$\text{and } \frac{dw}{d\xi} = 0 \ \text{ at } \xi = 0.5.$$

Applying the boundary conditions on the assumed polynomial a set of linear simultaneous equations is obtained. Solving this set of equations the coefficients (C_i) of the polynomial can be determined directly. It is worth pointing out that depending on the location of the support (a) and stiffness of the springs, coefficients of the polynomial would change and different start functions would be obtained. The higher-order functions are generated through a numerical implementation of the Gram - Schmidt orthogonalization procedure. To cater to the need of the numerical scheme, all the functions are described numerically at some suitably selected Gauss points.

Substitution of the complete energy expressions and approximate displacement field in Eq. (1) gives the set of system governing equations in matrix form,

$$[K]\{d\} = \{f\} \tag{8}$$

$[K]$, $\{d\}$ and $\{f\}$ are stiffness matrix, vector of unknown coefficients and load vector, respectively. The different elements of the stiffness matrix and the load vector are given below.

$$\begin{aligned} k_{ij} &= \frac{EI}{L^3} \sum_{i=1}^{nw} \sum_{j=1}^{nw} \int_0^1 \frac{d^2\phi_i}{d\xi^2} \frac{d^2\phi_j}{d\xi^2} d\xi \\ &+ \frac{1}{L} \sum_{i=1}^{nw} \sum_{j=1}^{nw} \int_0^1 N_x(\xi) \frac{d\phi_i}{d\xi} \frac{d\phi_j}{d\xi} d\xi \end{aligned} \tag{9}$$

$$f_j = \sum_{j=1}^{nw} P\phi_j \,|_{\xi=0} + \sum_{j=1}^{nw} P\phi_j \,|_{\xi=1} \tag{10}$$

The stiffness matrix $[K]$ contains the axial force distribution (N_x) whose values are not known a priori within its elements. Hence at the first step of the iteration N_x is assumed as zero and the unknown coefficients are calculated on the basis of this assumption. From the computed coefficient values the transverse displacement (w) of the beam are determined and from this displacement field the axial strain values along the beam are computed. Correspondingly, the stress and axial forces are calculated and process is repeated with new values of axial force. The calculated displacement values are compared with those from the previous iteration. If the error is outside the predefined limit, the process is repeated with new values of N_x, until error becomes less than the specified value and convergence is achieved. The modification of axial force distribution values are achieved through the following expression, $\{N_x\} = \{N_x\}_{old} + \lambda(\{N_x\} - \{N_x\}_{old})$, where λ is the relaxation parameter. Once convergence is achieved for a particular load step, an increment is provided to the concentrated load and the next load step starts by again initializing the axial force distribution to zero.

3.2 Dynamic Analysis

The governing set of equations for the dynamic analysis is derived following Hamilton's principle, which is expressed as,

$$\delta\left(\int_{\tau_1}^{\tau_2} (T - U)\, d\tau \right) = 0 \tag{11}$$

According to Hamilton's principle a dynamic system can be characterized by two energy functionals, kinetic energy and potential energy. In the mathematical expression T and U represent the total kinetic energy of the system and total strain energy stored in the system. The expression for total kinetic energy (T) is given by,

$$T = \frac{1}{2} \rho\, b\, t\, L \int_0^1 \left(\frac{\partial w}{\partial \tau} \right)^2 d\xi \tag{12}$$

where, ρ is density of the beam.

The dynamic displacement, $w(\xi, \tau)$, is assumed to be separable in space and time and can be approximately represented by finite linear combinations of orthogonal admissible functions and a new set of unknown coefficients c_i as,

$$w(\xi, \tau) = \sum_{i=1}^{nw} c_i \, \phi_i \, e^{i\omega\tau} \tag{13}$$

Here, ω is the natural frequency of the system and $\{c\}$ represents the eigenvectors in matrix form which indicates the contribution of the individual space functions in a particular vibration frequency mode. The spatial functions, $\phi_i(\xi)$, are identical to those for the static analysis and are known completely. Substitution of the kinetic (T) and strain energy (U) expressions along with the dynamic displacement field gives the governing equation of the dynamic system in the following form.

$$-\omega^2 [M]\{c\} + [K]\{c\} = 0 \tag{14}$$

where, $[M]$ is the mass matrix and its different elements are provided below.

$$M_{ij} = \rho\, b\, t\, L \sum_{j=1}^{nw} \sum_{i=1}^{nw} \int_0^1 \phi_i \, \phi_j \, d\xi \tag{15}$$

The solution of the standard eigen value problem of Eq. (14) is obtained numerically through IMSL routines. The square roots of the calculated eigen values represent the free vibration frequencies of the beam at the statically deflected configuration. The plot of these frequencies against the corresponding amplitudes in non-dimensional plane represents the backbone curve of the system.

4. RESULTS AND DISCUSSION

The present work has the objective of studying free vibration of simply supported overhang beam under concentrated loading, both experimentally and theoretically. For the theoretical analysis, the number of functions (nw) for the transverse displacement (w) is taken as 12, whereas, the number of Gauss points is taken as 24. The tolerance value of the error limit (ε) for the numerical iteration scheme is taken as 0.01% and the relaxation parameter (λ) is 0.50. Experiment is carried out for a beam of following dimensions: L = 0.275 m, b = 0.05266 m, t = 0.00228 m and a = 0.05 m. The material of the beams is mild steel.

Table 1: Natural frequencies of simply supported overhang beam under different load levels

Sl. No.	Load (Kg.)	Natural Frequency (Hz.)		
		1	2	3
1	W_P	44	82	152
2	W_P + 0.200	36	80	156
3	W_P + 0.500	28	78	152
4	W_P + 1.000	26	76	154
5	W_P + 2.000	36	80	152

The natural frequencies for the first three modes of vibration of the system are read from the oscilloscope and presented corresponding to different load levels in Table 1, where, W_P refers to the weight of the pan. The natural frequency of the beam under no loading condition is obtained as 108 Hz. However, the theoretical analysis provides the value as 149.9589 Hz. It can be seen that the theoretical and experimental results differ from each other. The difference can be attributed to insufficiency in replicating the boundary conditions of the system.

The theoretical dynamic behaviour of the simply supported overhang beam is shown graphically as the backbone curves for the first three modes in the dimensionless amplitude-frequency plane. The ratio of the maximum beam deflection to beam thickness is taken as the dimensionless amplitude (w_{max}/t) while the dimensionless frequency (ω_1/ω) is obtained by normalizing the nonlinear frequency (ω) with the corresponding fundamental linear frequency (ω_1).

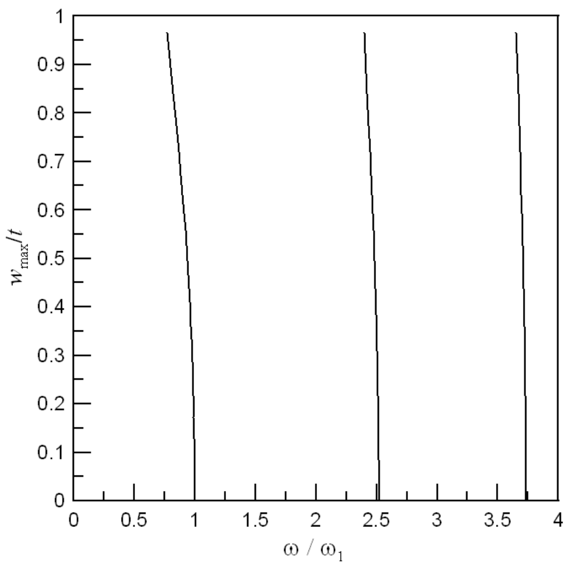

Fig 5. Backbone curve for simply supported overhang beam.

Figure 5 shows the backbone curve corresponding to the first three modes of vibration for a simply supported overhang beam under transverse concentrated loading at the extreme ends of the beam. The modeshapes for the vibrating system can be determined from the eigenvectors corresponding to the eigenvalues. The modeshape plots for the first three vibration modes of the beam under point loading are shown in figure 6. Changes in the first modeshape due to change in location of the support has also been studied and presented in Fig. 7. The effect of variation of overhang lengths is quite clear from the figure. From the figure it is clear that for a/L = 0.20 and 0.25 the maximum vibration amplitude occurs at the extremities of the beam; whereas, for the other two cases it occurs at the mid-point of the span. It can be said that as the overhang length increases, i.e., as the two supports move closer to each other the maximum vibration amplitude takes place at the two ends of the beam.

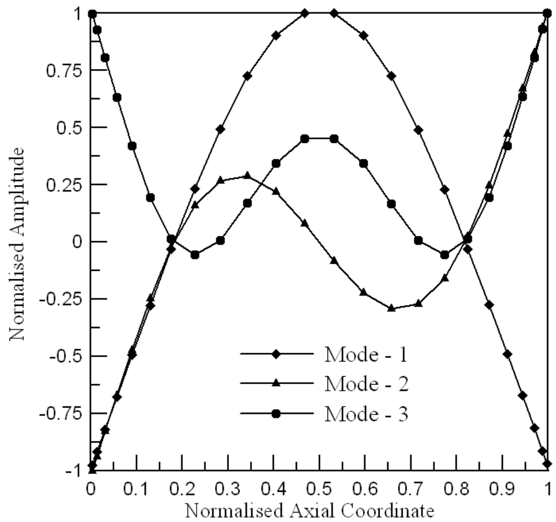

Fig 6. Mode shape plots for simply supported overhang beam under concentrated loading at its ends.

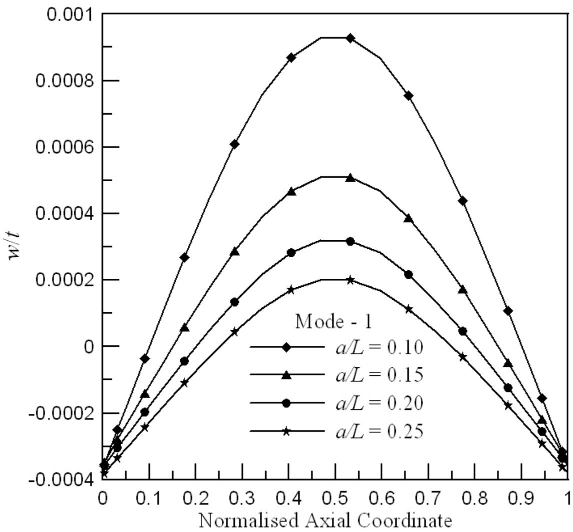

Fig 7. Variation in first modeshape for change in support location.

5. CONCLUSION

The present paper undertakes an experimental and theoretical dynamic analysis of a simply supported overhang beam under transverse concentrated loading at the ends. A set up simulating simply supported boundary conditions is developed and experimentations are carried out. The mathematical formulation is based on energy methods and axial stretching is incorporated through an iterative procedure involving successive relaxation of axial forces. Solution of the governing set of equations is achieved through IMSL routines.

6. ACKNOWLEDGEMENT

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8. NOMENCLATURE

Symbol	Meaning	Unit
a	Length of overhang	m
b	Width of beam	m
Ci	Coefficients of Polynomial	-
c_i	Unknown coefficients for dynamic analysis	-
d_i	Unknown coefficients for static analysis	-
E	Elastic modulus of beam material	N/m^2
F	Force at support locations	N
$\{f\}$	Load vector	N
I	Moment of inertia of beam cross-section	m^4
$[K]$	Stiffness matrix	N/m
k	Stiffness of springs	N/m
L	Length of beam	m
l	Length between two simple supports	m
$[M]$	Mass matrix	kg
N_x	Axial force distribution	N
nw	Numbers of orthogonal functions	-
P	Magnitude of concentrated load	N
T	Kinetic energy of the system	N-m/s
t	Thickness of beam	m
U	Strain energy stored in the system	N-m/s
V	Potential energy of external forces	N-m/s
w	Transverse displacement field	m
x	Axial coordinate	m
$\phi_i(\xi)$	Set of orthogonal functions	-
ω	Natural frequency	Hz.
τ	Time coordinate	s
ξ	Normalized axial coordinate	-
δ	Variational operator	-
ρ	Density of beam	Kg/m^3
λ	Relaxation parameter	-

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